

## Geometry 3-5 Study Guide: Slopes of Lines (pp 182-184)

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**Attendance Problems.** Find the value of  $m$ . Write your answer as an integer or as fraction in reduced terms.

1.  $m = \frac{7-5}{8-3}$

2.  $m = \frac{(-3)-6}{5-(-1)}$

3.  $m = \frac{4-(-4)}{2-2}$

4.  $m = \frac{-3+3}{1-6}$

- I can find the slope of a line.
- I can use slopes to identify parallel and perpendicular lines.

Vocabulary		
rise	run	slope

**Common Core:** CC.9-12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of line parallel or perpendicular to a given line that passes through a given point).

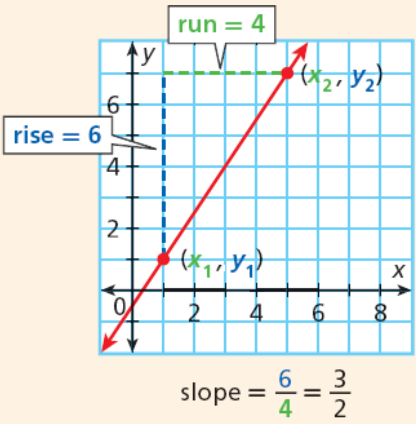
### Slope Song (Turkey in the Straw)

Slope is rise over run as we all know.  
With the Y's on the top and the X's below. Subtract the terms to get it right.  
Simplify last for a wonderful sight.

(Chorus)

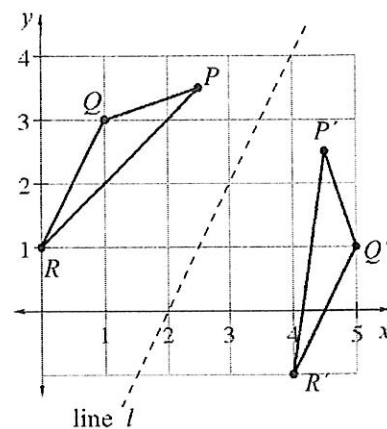
Rise over run, Y's over X.  
Rise over run, Y's over X.  
Subtract the terms to get it right.  
Simplify last for a wonderful sight!

## Slope of a Line

DEFINITION	EXAMPLE
The <b>rise</b> is the difference in the y-values of two points on a line.	 <p>run = 4</p> <p>rise = 6</p> <p>slope = <math>\frac{6}{4} = \frac{3}{2}</math></p>
The <b>run</b> is the difference in the x-values of two points on a line.	
The <b>slope</b> of a line is the ratio of the rise to run. If $(x_1, y_1)$ and $(x_2, y_2)$ are any two points on a line, the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$ .	

Today, you will learn about the relationships between an object and its image that will help you predict the image's position. These relationships are described using algebra and just one example of connections between new concepts you are studying in geometry and the math you learned in previous courses.

5.  $\triangle PQR$  was reflected across line  $l$  to form  $\triangle P'Q'R'$ . Describe the relationship of the original triangle and its image to the line of reflection. Specifically, how far away is each triangle from its line of reflection? What do you notice about the location of the image relative to the angle of reflection and the original triangle?



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Use the following information to answer questions 6 & 7: In problem 5, you made some observations about reflections. Slope can help reveal more about transformations such as reflections.

6. Begin by graphing the equation

$$y = \frac{3}{5}x - 4.$$
 Using tracing paper to

translate the graph of  $y = \frac{3}{5}x - 4$  up

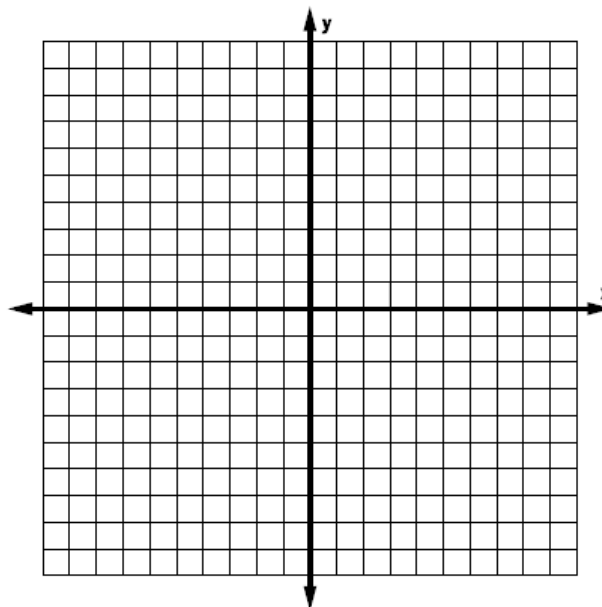
5 units. Write the equation of the

resulting image. What is the

relationship between  $y = \frac{3}{5}x - 4$  and

its image? How do their slopes

compare?



7. Now use tracing paper to rotate

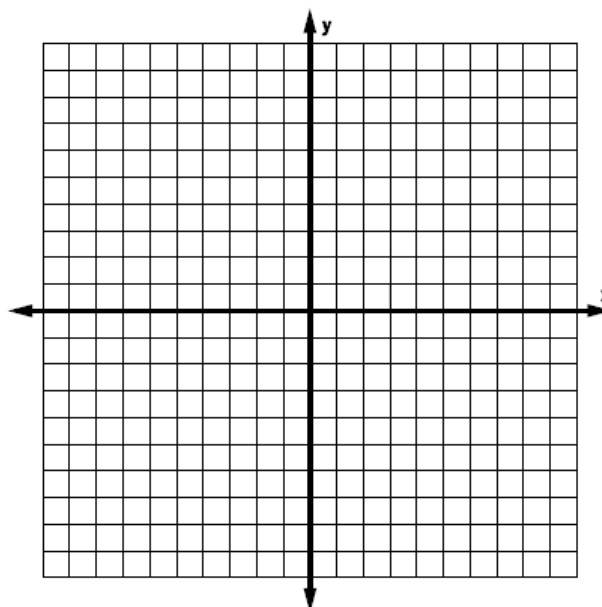
$$y = \frac{3}{5}x - 4$$
 clockwise  $90^\circ$  ( $\curvearrowright$ ) about

(0, 0). Write the equation of the result.

Describe the relationship between

$$y = \frac{3}{5}x - 4$$
 and this new image. How

do the slopes compare?



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**Use the following information to answer questions 8-12: Do you think all perpendicular slopes are related the way you found in problem 7? Investigate this idea by drawing three different slopes on graph paper on sketchpad. It will work better if you select “snap points” under the graph menu.**

**8.** Construct a line with a slope of 1. Rotate the line  $90^\circ$ , either clockwise or counterclockwise. Find the slope of the new line.

**9.** Construct a line with a slope of  $\frac{1}{3}$ . Rotate the line  $90^\circ$ , either clockwise or counterclockwise. Find the slope of the new line.

**10.** Construct a line with a slope of  $-\frac{3}{4}$ . Rotate the line  $90^\circ$ , either clockwise or counterclockwise. Find the slope of the new line. Write your answer as a fraction.

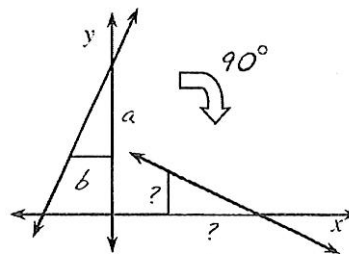
**11.** How does the slope of each rotated line compare to the slope of its original line? Share any patterns with your table-mates.

**12.** Use patterns from the work you have done to describe the general relationship of the slopes of perpendicular lines. That is, if you have two perpendicular lines and know the slope of one, how can you find the slope of the other?

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**Slopes of Perpendicular lines.** Use the following information to answer questions 14 & 15: Two lines are perpendicular whenever one line can be rotated  $90^\circ$  onto the other. However, a rotation does not only move points along the line--it moves all the points on the graph! Therefore, the relationship of the slopes of perpendicular lines can be demonstrated as true by rotating a non-special (not vertical or horizontal) along with its slope.



However, to prove something mathematically, you must be able to explain it in all cases, not just particular numbers. To prove the relationship between perpendicular slopes, Sabrina drew the picture shown.

14. Use Sabrina's drawing to explain the slope of the perpendicular line must always be  $-\frac{b}{a}$  if neither  $a$  nor  $b$  is zero.

15. What if the original line has a slope of 0? Explain what happens to a line with slope 0 if it is rotated  $90^\circ$ , and what the slope of the perpendicular line would be.

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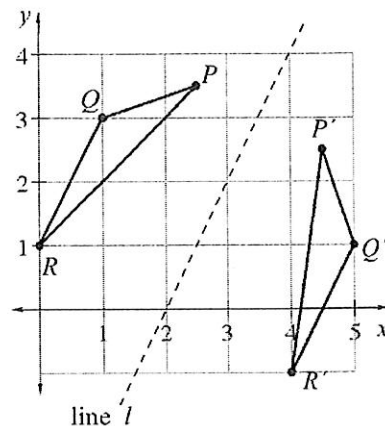
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Use the following information to answer questions 16-18. Now that you know more about the slope of parallel and perpendicular lines, revisit the reflection from problem 5 and confirm the relationship using slope.

16. In the diagram, the coordinates are  $P(2.5, 3.5)$ ,  $Q(1, 3)$ ,  $R(0, 1)$  and  $P'(4.5, 2.5)$ ,  $Q'(5, 1)$ ,  $R'(4, -1)$ . (If you want, you can graph these on your own paper or use the graph provided.)

Use your ruler to draw three dashed lines:  $\overline{PP'}$ ,  $\overline{QQ'}$ , &  $\overline{RR'}$ .

What is the relationship of these dashed lines? Use your knowledge of slope to verify your observation.



17. Now focus on the relationship between the lines of reflection and each of the segments connecting a point with its image. What do you notice about lengths and angles. Be as specific as you can. Use what you know about reflections to explain why your observations must be true.

18. Use slope to confirm that the line of reflection is perpendicular to the line segments connecting each original point and its image.

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19. Evan has graphed the point  $(5, 7)$  and he wants to reflect it over the line  $y = -\frac{2}{5}x + 6$ . He predicts that the reflected point will have coordinates  $(2, 2)$ .

*Without graphing*, can you confirm his answer or show that he cannot be correct?

20. Suppose the equation for line A is  $y = \frac{6}{5}x - 10$ . Line A is parallel to line B, which is perpendicular to line C. If line D is perpendicular to line C and perpendicular to line E, what is the slope of line E? Justify your conclusion.

21. **Slopes of Perpendicular Lines.** Summarize what you have learned in today's lesson. Be sure to explain the relationship between the slopes of perpendicular lines and describe how to get the slope of one line when you know the slope of the line perpendicular to it.

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**Use the following information to answer questions 22 & 23: Use what you know about the slopes of parallel & perpendicular lines to find the equation of a line that that would meet the criteria given.**

**22.** Find the equation of the line that goes through the point (0, -3) and is perpendicular to the line  $y = -\frac{2}{5}x + 6$ .

**23.** Find the equation of the line that is parallel to the line  $-3x + 2y = 10$  goes through the point (0, 7).

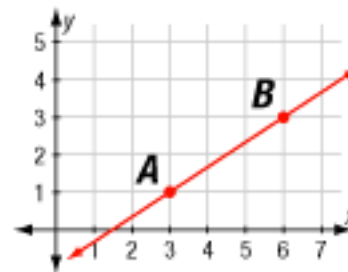


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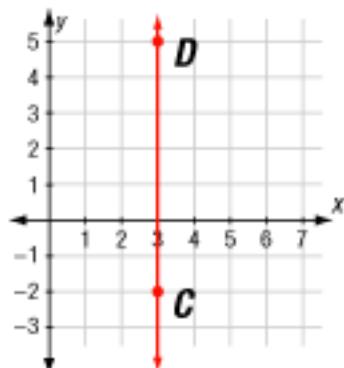
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**Video Example 1.** Find the slope of each line.

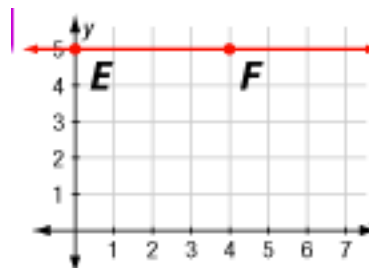
**A.** AB.



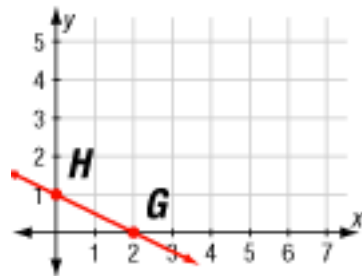
**B.** CD.



**C.** EF.



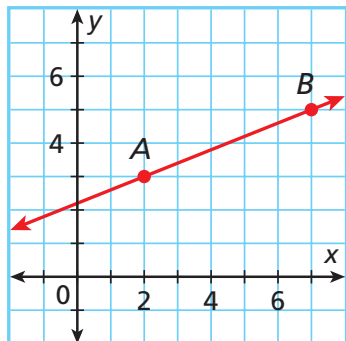
**D.** HG



## 1 Finding the Slope of a Line

Use the slope formula to determine the slope of each line.

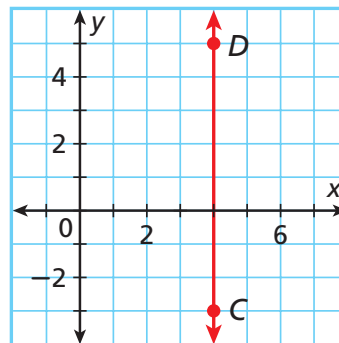
**A**  $\overleftrightarrow{AB}$



Substitute  $(2, 3)$  for  $(x_1, y_1)$  and  $(7, 5)$  for  $(x_2, y_2)$  in the slope formula and then simplify.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{7 - 2} = \frac{2}{5}$$

**B**  $\overleftrightarrow{CD}$



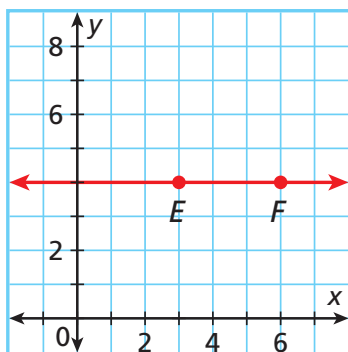
Substitute  $(4, -3)$  for  $(x_1, y_1)$  and  $(4, 5)$  for  $(x_2, y_2)$  in the slope formula and then simplify.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-3)}{4 - 4} = \frac{8}{0}$$

The slope is undefined.

Use the slope formula to determine the slope of each line.

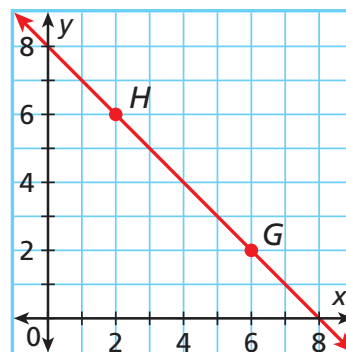
**C**  $\overleftrightarrow{EF}$



Substitute  $(3, 4)$  for  $(x_1, y_1)$  and  $(6, 4)$  for  $(x_2, y_2)$  in the slope formula and then simplify.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{6 - 3} = \frac{0}{3} = 0$$

**D**  $\overleftrightarrow{GH}$



Substitute  $(6, 2)$  for  $(x_1, y_1)$  and  $(2, 6)$  for  $(x_2, y_2)$  in the slope formula and then simplify.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 2}{2 - 6} = \frac{4}{-4} = -1$$

**Q:** How do the geometry teacher and track coach wake up their son?

**A:** It's time to rise and run!

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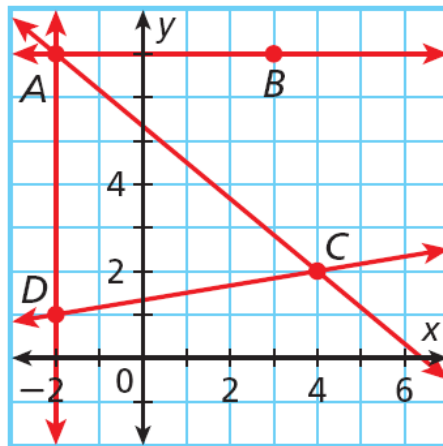
**Example 1.** Find the slope of each line.

A. AB

B. AC

C. AD

D. CD



### Remember!

A fraction with zero in the denominator is undefined because it is impossible to divide by zero.

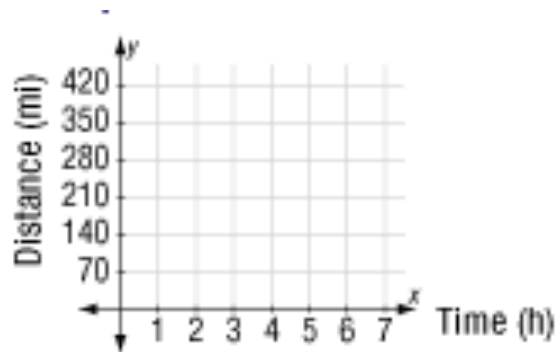
Summary: Slope of a Line			
Positive Slope	Negative Slope	Zero Slope	Undefined Slope

One interpretation of slope is a **rate of change**. If  $y$  represents miles traveled and  $x$  represents time in hours, the slope gives the rate of change in miles per hour.

A person with one watch knows what time it is; a person with two watches is never sure. *Proverb*

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**Video Example 2.** Missy is driving from New Orleans, Louisiana, to Dallas, Texas. At 2 pm, she is 140 miles from New Orleans. At 5 pm, she is 350 miles from the New Orleans. Graph the line that represents the distance from New Orleans at a given time. Find and interpret the slope of the line.



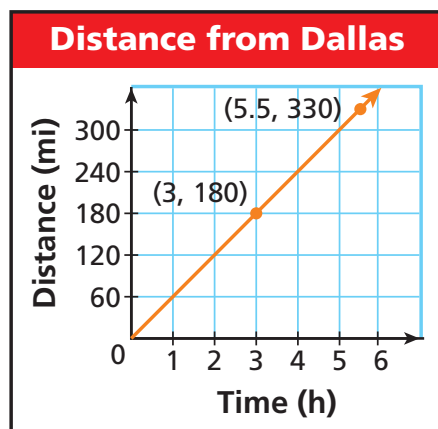
### 2 Transportation Application

Tony is driving from Dallas, Texas, to Atlanta, Georgia. At 3:00 P.M., he is 180 miles from Dallas. At 5:30 P.M., he is 330 miles from Dallas. Graph the line that represents Tony's distance from Dallas at a given time. Find and interpret the slope of the line.

Use the points  $(3, 180)$  and  $(5.5, 330)$  to graph the line and find the slope.

$$m = \frac{330 - 180}{5.5 - 3} = \frac{150}{2.5} = 60$$

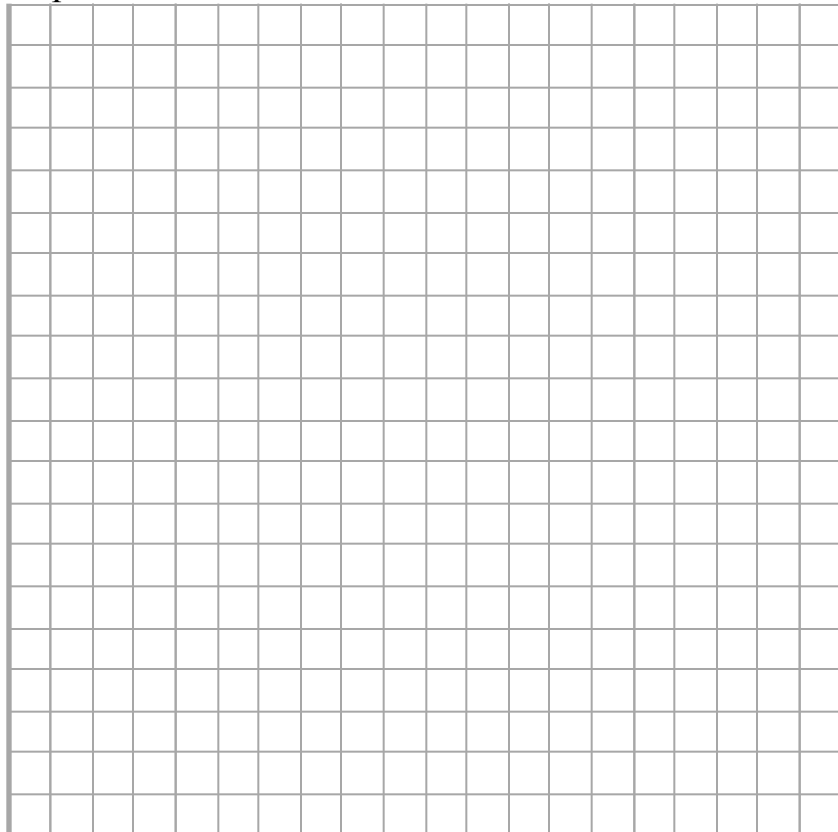
The slope is 60, which means he is traveling at an average speed of 60 miles per hour.



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**Example 2.** Justin is driving from home to his college dormitory. At 4:00 p.m., he is 260 miles from home. At 7:00 p.m., he is 455 miles from home. Graph the line that represents Justin's distance from home at a given time. Find and interpret the slope of the line.



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